Korean J. of Chem. Eng., 13(1), 15-20 (1996)

THE ONSET OF DOUBLE-DIFFUSIVE CONVECTION IN A STABLY STRATIFIED FLUID LAYER HEATED FROM BELOW

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(Received 3 February 1995 • accepted 3 November 1995)

Abstract—The time of the onset of double-diffusive convection in time-dependent, nonlinear temperature fields is investigated theoretically. The initially quiescent horizontal fluid layer with a uniform solute gradient experiences ramp heating from below, but its stable solute concentration is to reduce thermal effects which invoke convective motion. The related stability analysis is conducted on the basis of the propagation theory. Under the linear stability theory the thermal penetration depth is used as a length scaling factor and the linearized perturbation equations of similarity transform are solved numerically. The resulting correlations of the critical time to mark the onset of regular cells are derived as a function of the thermal Rayleigh and the solute Rayleigh numbers. The predicted stability criteria are apparently consistent with existing experimental results for aqueous solution of sodium chloride.

Key words: Double-diffusive Convection, Propagation Theory, Thermal Penetration Depth, Thermal Rayleigh Number, Solute Rayleigh Number

INTRODUCTION

Buoyancy-driven convection in double-diffusive systems has been studied extensively in connection with wide engineering situations such as crystal growth processing, solar ponds and natural gas storage tanks [Chen and Johnson, 1984; Ostrach, 1983; Turner, 1973]. Recently, the role of convection in growing semiconductor crystals has been an active research topic, since convective motion is deleterious for manufacturing high-grade crystals. But the inherent complexity in practical systems makes it very difficult to predict the stability criteria by which the effect of natural convection is determined in the process design. This comes from the fact that the solute concentration and temperature profiles are nonlinear and time-dependent.

When an initially motionless, stable concentration-stratified fluid layer is placed between two horizontal plates with its bottom boundary heated suddenly, natural convection will set in at a certain time, depending on both the thermal Rayleigh number and the solute Rayleigh number [Nield, 1967]. Therefore, it becomes an important problem to predict the critical time to mark the onset of convective motion. For this purpose, several theoretical models have been used in deep-pool systems of high Rayleigh numbers: the amplification theory [Foster, 1965], energy method [Wankat and Homsy, 1977], stochastic model [Jhaveri and Homsy, 1982], and propagation theory [Choi et al., 1986]. Even though they are all good models, the present double-diffusive convection has been analysed only by the amplification theory. The amplification theory has been quite popular, but it involves difficulties in deciding the initial conditions and also choosing the growth factor to determine the onset time. Comparing with other methods, the energy method predicts the onset time of buoyancy-driven convection as lower bound. And the stochastic model involves some arbitrariness. But the propagation theory which we have developed decides deterministically the criteria to mark the onset time by using the thermal penetration depth as a length scaling factor and transforming the linearized perturbation equations. Predicted values resulting from the principle of exchange of stabilities have been consistent with most of experimental data in systems of laminar forced convection [Ahn and Choi, 1988], internal heat generation [Choi et al., 1992] and also fluid-saturated porous layers [Yoon and Choi, 1989]. Therefore, the stability analysis based on the propagation theory will be extended to the present problem of the onset of double-diffusive convection caused by ramp heating from below.

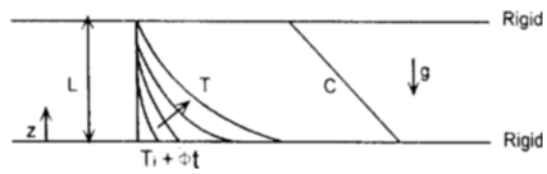

Fig. 1. Schematic diagram of the system.

STABILITY ANALYSIS

1. Mathematical Formulation

The problem considered here is a horizontal fluid layer confined between two rigid boundaries separated by a distance L, as shown in Fig. 1. The fluid layer is initially quiescent at a constant temperature T_i and stably stratified by a uniform solute-concentration gradient. At the time $t=0$ the lower surface of the fluid layer is heated suddenly with a constant temporal rate ϕ. Therefore the bottom temperature increases linearly with time. For high-ϕ systems natural convection will set in at a certain time due to buoyancy forces. Under this ramp-heating condition

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the density variation of fluid is assumed to follow the usual equation of state [Nield, 1967]:

$$\rho=\rho_0[1-\beta(T-T_0)+\gamma(C-C_0)] \quad (1)$$

where ρ, T, C, β and γ represent the fluid density, the temperature, the solute concentration, the volumetric thermal expansion coefficient, and the volumetric solute expansion coefficient, respectively. The subscript 0 denotes the basic state.

The important parameters to characterize the onset of motion in the present system are the thermal Rayleigh number Ra, the solute Rayleigh number Rs, the Prandtl number Pr and the Lewis number Le, defined by

$$Ra=\frac{g\beta\phi L^5}{\alpha^2\nu},\ Rs=\frac{g\gamma L^3\Delta C}{\alpha_s\nu},\ Pr=\frac{\nu}{\alpha},\ Le=\frac{\alpha}{\alpha_s}$$

where g, α, ν, ΔC and α_s denote the gravitational acceleration, the thermal diffusivity, the kinematic viscosity, the concentration difference and the solute diffusivity, respectively. Under the linear stability theory, the nondimensionalized conservation equations are constituted as follows [Kaviany, 1984a; Kaviany and Vogel, 1986]:

$$\left(\frac{1}{Pr}\frac{\partial}{\partial\tau}-\nabla^2\right)\nabla^2 w_1=\nabla_1{}^2\theta_1-\frac{1}{Le}\nabla_1{}^2\Psi_1 \quad (2)$$

$$\frac{\partial\theta_1}{\partial\tau}+Ra\ w_1\frac{\partial\theta_0{}^*}{\partial z}=\nabla^2\theta_1 \quad (3)$$

$$\frac{\partial\Psi_1}{\partial\tau}+Rs\ w_1\frac{\partial\Psi_0}{\partial z}=\frac{1}{Le}\nabla^2\Psi_1 \quad (4)$$

$$\frac{\partial\theta_0{}^*}{\partial\tau}=\frac{\partial^2\theta_0{}^*}{\partial z^2} \quad (5)$$

$$\frac{\partial\Psi_0}{\partial\tau}=\frac{1}{Le}\frac{\partial^2\Psi_0}{\partial z^2} \quad (6)$$

where ∇^2 is the three-dimensional Laplacian, and $\nabla_1{}^2$ is the horizontal one with respect to x and y. Here z, τ, w_1, $\theta_0{}^*$, θ_1, Ψ_0, and Ψ_1 are the dimensionless vertical distance, time, perturbed vertical velocity, basic temperature, perturbed temperature, basic concentration, and perturbed concentration, respectively. Each variable has been nondimensionalized by using L, L^2/α, α/L, $\phi L^2/\alpha$, $\nu\alpha/g\beta L^3$, ΔC and $\nu\alpha_s/g\gamma L^3$, respectively. The proper boundary conditions are

$$w_1=\frac{\partial w_1}{\partial z}=\theta_1=\Psi_1=0 \qquad \text{for } z=0 \text{ and } z=1 \quad (7)$$

$$\theta_0{}^*=\tau \qquad \text{at } z=0 \quad (8a)$$

$$\theta_0{}^*=0 \qquad \text{at } z=1 \quad (8b)$$

$$\frac{\partial\Psi_0}{\partial z}=0 \qquad \text{for } z=0 \text{ and } z=1 \quad (9)$$

Eq. (7) satisfies the conditions of no fluctuation of perturbed quantities at rigid boundaries. The boundary conditions (8) and (9) come from Kaviany and Vogel's [1986].

Through the method of the separation of variables, the Graetz-type solution for the basic temperature is easily obtained as

$$\theta_0{}^*=(1-z)\tau+\sum_{n=1}^{\infty}\frac{-2}{(n\pi)^3}\sin(n\pi z)[1-\exp(-n^2\pi^2\tau)] \quad (10)$$

Since this exact solution involves mathematical difficulties in the present deep-pool system of small τ wherein the similarity solution exists. For a deep-pool system, the basic temperature $\theta_0{}^*$ is transformed with a similarity variable, $\eta=z/\sqrt{\tau}$ by using the relation of

$$\theta_0{}^*=\tau\theta_0(\eta) \quad (11)$$

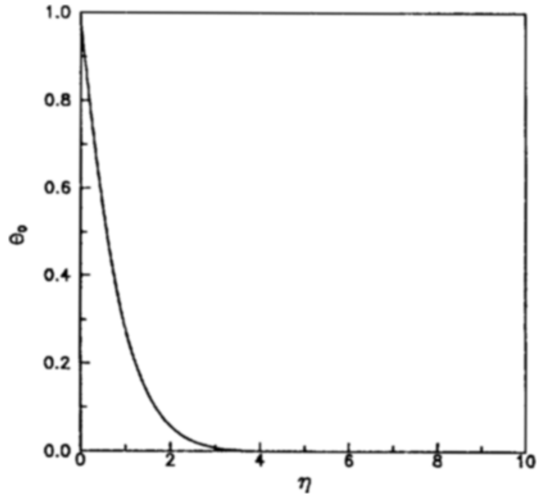

Fig. 2. Base temperature profile.

Then a new set of equations are generated from Eqs. (5), and (8):

$$\frac{d^2\theta_0}{d\eta^2}+\frac{1}{2}\eta\frac{d\theta_0}{d\eta}-\theta_0=0 \quad (12)$$

$$\theta_0=1 \text{ at } \eta=0 \quad (13a)$$

$$\theta_0=0 \text{ for } \eta\to\infty \quad (13b)$$

The solution of θ_0 is obtained numerically, as shown in Fig. 2. This similarity solution agrees well with the exact one for $\tau\leq 0.1$.

At the initial state for the stable concentration-stratified fluid layer, the dimensionless concentration field satisfying Eq. (6) will be linear as shown in Fig. 1. The effect of the linear distribution is expected to stabilize the fluid layer. Under the boundary condition of Eq. (9), the exact solution for the dimensionless concentration is obtained as follows,

$$\Psi_0=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{4}{(2n-1)^2\pi^2}\cos[(2n-1)\pi z]\exp\left[\frac{-(2n-1)^2\pi^2\tau}{Le}\right] \quad (14)$$

Therefore the base density field satisfying the equation of state can be defined as

$$\rho^*=-\theta_0{}^*+\frac{Rs}{LeRa}\Psi_0 \quad (15)$$

where ρ^* denotes the nondimensionalized base density scaled by $\rho_0\phi L^2\beta/\alpha$. The resultant variation of the profile of the base density with respect to time is shown in Fig. 3 where maximum magnitude of density locates within the fluid layer. The density distribution for this system is quite similar to that for the internally heat-generating one which has been analysed by the propagation theory [Choi et al., 1992].

2. Propagation Theory

For a given Ra, Pr, Rs and Le the time to mark the onset of convective motion is to be found under the principle of exchange of stabilities from Eqs. (2)-(4), subjected to the boundary condition (7). Even though the initially stratified density field may

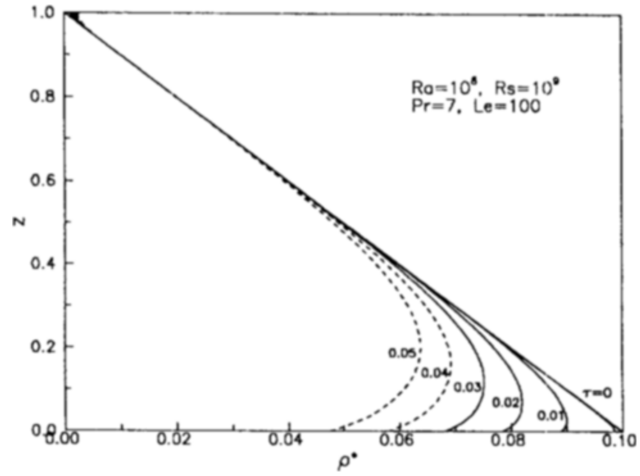

Fig. 3. Base density profiles with respect to time for $Ra=10^8$ and $Rs=10^8$.

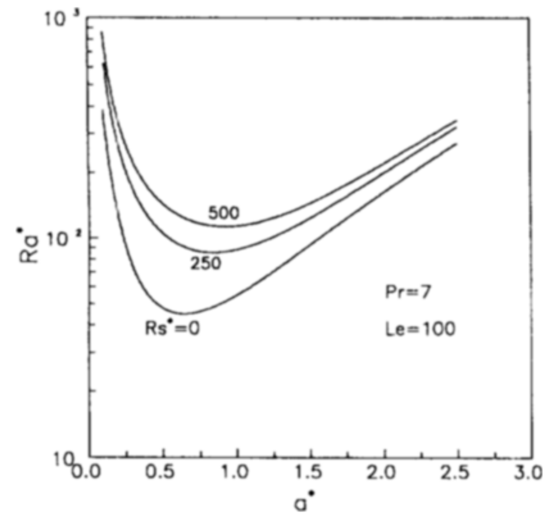

Fig. 4. Neutral stability curves for Pr=7 and Le=100.

reduce the magnitude of the generated disturbances, the disturbances are to be generated continuously. Therefore the density distribution for molecular diffusion of heat and salt in water is time-dependent. This is a formidable task to obtain quantitative results for the onset time of the double-diffusive convection, and therefore, we will employ the propagation theory we have developed. Since there is no lateral boundary in the present system, it is assumed that the horizontal variations of disturbances at the onset time are represented by the dimensionless wave numbers, a_x and a_y, as follows:

$$(w_1, \theta_1, \Psi_1)=[w_1^*(\tau, z), \theta_1^*(\tau, z), \Psi_1^*(\tau, z)]\exp[i(a_x+a_y)] \quad (16)$$

where i is the imaginary number. As the buoyancy effects are confined in the thermal penetration depth, the length scale components are rescaled by the dimensionless thermal penetration depth δ having the value of $\theta_0=0.01$. Fig. 2 shows that $\delta=3.8\sqrt{\tau}$ for $\tau\leq 0.1$ in the present system. By using the relation of $\delta\propto\sqrt{\tau}$ [Howard, 1964] amplitude functions are transformed as

$$[w_1^*(\tau, z), \theta_1^*(\tau, z), \Psi_1^*(\tau, z)]=[\tau w^*(\eta), \theta^*(\eta), \Psi^*(\eta)] \quad (17)$$

Now, the new amplitude functions w^*, θ^* are Ψ^* dependent on η only. Then, for the uniform concentration gradient we can get the following set of stability equations in terms of the horizontal wave number $a=\sqrt{a_x^2+a_y^2}$, from Eqs. (2)-(4):

$$(D^2-a^{*2})^2w^*+\frac{1}{2Pr}(\eta D^3-a^{*2}\eta D+2a^{*2})w^*-a^{*2}\theta^*+\frac{1}{Le}a^{*2}\psi^*=0 \quad (18)$$

$$\left(D^2+\frac{1}{2}\eta D-a^{*2}\right)\theta^*=Ra^*w^*D\theta_0 \quad (19)$$

$$\left(D^2+\frac{Le}{2}\eta D-a^{*2}\right)\Psi^*=-LeRs^*w^* \quad (20)$$

with boundary conditions:

$$w^*=Dw^*=\theta^*=\Psi^*=0 \text{ for } \eta=0 \text{ and } \eta\to\infty \quad (21)$$

where $a^*=a\sqrt{\tau}$, $Ra^*=Ra\sqrt{\tau^5}$, $Rs^*=Rs\tau^2$ and $D=d/d\eta$. These equations involve time-dependent properties implicitly. It is assumed that a^*, Ra^* and Rs^* are all eigenvalues and the principle of exchange of stabilities is kept. This is essence of the propagation theory. For a given Pr, Le, a^* and Rs^* the minimum value of Ra^* will be found numerically.

3. Solution Method

In order to integrate the stability equations, Eqs. (18)-(20), trial values of the eigenvalue Ra^* and the boundary conditions D^3w^*, $D\theta^*$, and $D\Psi^*$ at $\eta=0$ are assumed properly for a given Pr, Le, a^* and Rs^*. Here, the values of Pr and Le are fixed at 7 and 100, respectively, in order to compare the present stability criteria with Kaviany and Vogel's [1986]. Since boundary conditions represented by Eq. (21) are all homogenous, the value of Dw^* at $\eta=0$ can be assigned arbitrarily. This procedure is based on the outward shooting method in which the boundary value problem is transformed into the initial value problem. The trial values, with together the four known conditions at the heated boundary, give all the information to make numerical integration. The integration based on the 4th-order Runge-Kutta method is performed from $\eta=0$ to a fictitious distance to satisfy the infinite boundary conditions. By using the Newton-Raphson iterations the trial values of Ra^*, D^3w^*, $D\theta^*$ and $D\Psi^*$ are corrected until the stability equations satisfy the infinite boundary conditions within the maximum relative tolerance of 10^{-8}. Then, by increasing the distance step by step, the above integration is repeated. Finally, the value of Ra^* is decided through extrapolation.

RESULTS AND DISCUSSION

With changing values of a^* and then Rs^*, the neutral stability curves are obtained for Le=100 which denotes the ratio of molecular diffusivity for heat to that for salt. And also, the Prandtl number is taken to be 7 for the present system involving double diffusion of heat and salt in water. The resulting neutral stability curves are shown in Fig. 4. According to the present theory it is considered that for a given Rs^* the minimum value of Ra^* on each curve of Ra^* and a^*, as shown in Fig. 5, characterizes the critical condition of convective motion. The present critical conditions to mark the onset of motion are represented by the following correlation within the bound of ±1% error:

$$Ra_c^*=45.0+0.64Rs_c^{*3/4} \text{ for } \tau_c\leq 0.1 \quad (22)$$

By using the relation of $Ra_c^*=Ra\sqrt{\tau_c^5}$ and $Rs_c^*=Rs\tau_c^2$, we can obtain the critical time τ_c for a given Ra and Rs. When the present

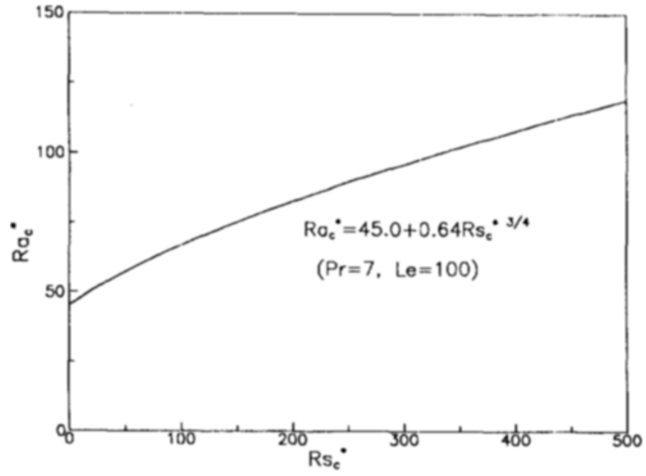

Fig. 5. Critical conditions of double-diffusive convection.

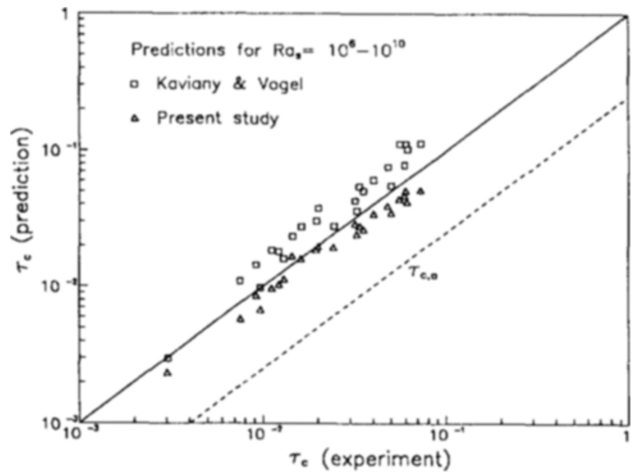

Fig. 7. Comparison of predicted values with experimental measurements.

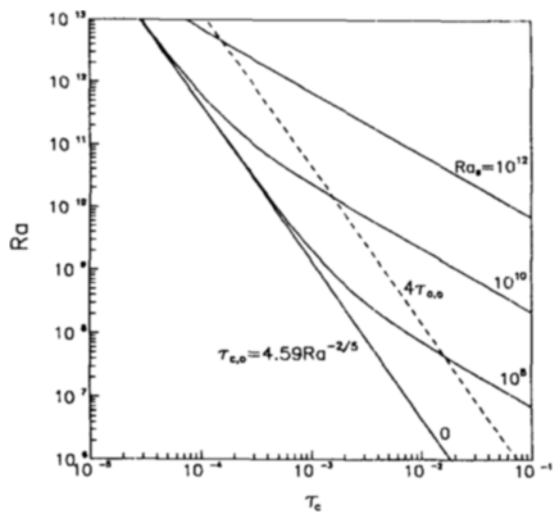

Fig. 6. Critical time with respect to Ra for Pr=7 and Le=100.

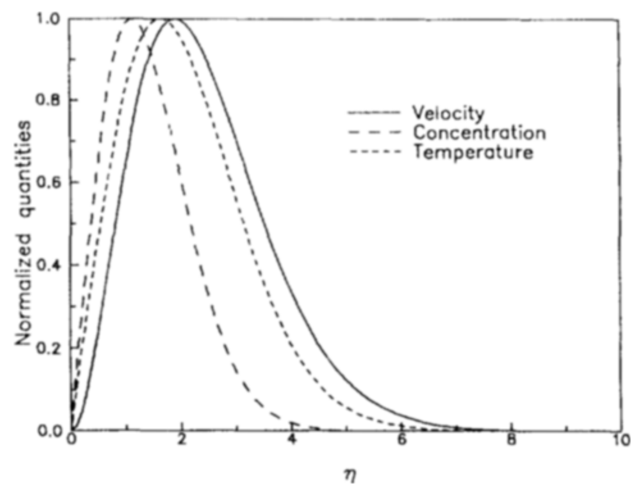

Fig. 8. Distribution of amplitude functions at Ra*=90 and Rs*=250.

correlation for Pr=7 and Le=100 is compared with the predictions of Kaviany and Vogel [1986], it is seen that the present predictions are almost one fourth of theirs based on the amplification theory. For detailed discussion the typical results are summarized in Fig. 6. In the figure the minimum bound of Ra*, i.e., $\tau_{c,o}=4.59\text{Ra}^{-2/5}$ corresponds to the case of the zero solute gradient of Rs=0. In this limiting case the Lewis number does not affect the critical time, and the present values of τ_c are about one fourth of Kaviany's results predicted by the amplification theory [Kaviany, 1984b]. It is thought that the most dangerous instabilities initiated at the time τ_c will grow to manifest themselves around the time $4\tau_c$. For $\tau_c<0.1$ the value of $4\tau_c$ represents manifest convection very well in other transient deep-pool systems [Lee et al., 1988; Yoon and Choi, 1989].

In the present deep-pool system the critical time becomes larger with increasing Rs, as shown in the figure. This means that the layer with large Rs requires the larger buoyancy force to induce convective motion because of more stable stratification. It is interesting that the effect of the stabilizing solute gradient begins to be noticeable at $\tau_c=4\tau_{c,o}$, considering the growth period of disturbances. This interpretation seems more reasonable than that of Kaviany and Vogel's by which the appreciable effect is seen at approximately Rs=0.1Ra [Kaviany and Vogel, 1986]. In Fig. 7 the present predictions of τ_c are compared with both experimental data and theoretical predictions of Kaviany and Vogel. The solid line means coincidence between the experimental and theoretical results. Their experimental data for Pr=7 and Le=100 range over Rs=10^8-10^{10}. For $\tau_c<0.1$ it is found that the present results are very reasonable in comparison with their experimental data. For ours are a little lower than experimental data, while Kaviany and Vogel's are somewhat higher than data points. It is interesting that our predictions for large Rs approach the experimental values more closely than those for Rs=0. All these studies justify to a certain degree that the onset of motion is characterized by regular cells. These results will be strictly valid only very close to the critical state, since the subsequent motion is often oscillatory in time [Nield, 1967]. Furthermore, for the base state of linear profiles the stability conditions can be represented by

$$\text{Ra}=\frac{\text{Rs}}{\text{Le}}+1708 \text{ for large } \tau_c \tag{23}$$

which will be the stability limit. Incorporating this limiting condi-

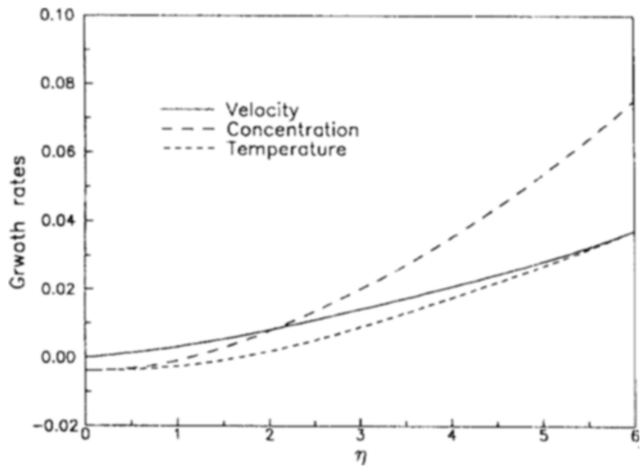

Fig. 9. Temporal growth rates for Ra = 10^8 and Rs = 10^8.

tion in Fig. 6, it may be loosely stated that these predictions cover the whole domain of present interest.

The typical amplitude distributions at $Rs^*=250$ and $Ra^*=90$ are featured in Fig. 8, as quantities normalized in terms of maximum magnitude. It is known that disturbances are mainly confined within the thermal penetration depth, i.e., $\eta=3.8$. In other cases the similar trends have been observed. Therefore, it seems certain that the thermal penetration depth is the proper length scale in many of deep-pool systems. From Eq. (17) the temporal growth rates of disturbance amplitudes at $\tau=\tau_c$ are obtained as follows:

$$\frac{1}{w_1^*}\frac{\partial w_1^*}{\partial\tau}\bigg|_{\tau=\tau_c}=\frac{1}{\tau_c}\left(1-\frac{\eta}{2w^*}Dw^*\right) \tag{24}$$

$$\frac{1}{\theta_1^*}\frac{\partial\theta_1^*}{\partial\tau}\bigg|_{\tau=\tau_c}=-\frac{1}{\tau_c}\left(\frac{\eta}{2\theta^*}D\theta^*\right) \tag{25}$$

$$\frac{1}{\Psi_1^*}\frac{\partial\Psi_1^*}{\partial\tau}\bigg|_{\tau=\tau_c}=-\frac{1}{\tau_c}\left(\frac{\eta}{2\Psi^*}D\Psi^*\right) \tag{26}$$

For $Pr=7$, $Le=100$, $Ra=10^8$ and $Rs=10^8$ the growth rates are obtained along the vertical distance, as shown in Fig. 9. This indicates that disturbances do not grow in the form of an exponential function with respect to τ. Considering the distributions shown in the figure, it is known that the amplitudes of both temperature and concentration disturbances near the bottom boundary are damped. It seems that this decrement of thermal and chemical potentials brings an increase in kinetic energy of disturbances in the outer region.

CONCLUSION

The onset of double-diffusive convection due to ramp heating in an initially stably stratified fluid layer with a uniform solute gradient has been analyzed deterministically by using the propagation theory, and the amplification of disturbances has been discussed qualitatively. In comparison with extant experimental data the present stability criteria look very reasonable. It appears apparent that in the range of $\tau_c\geq 4\tau_{c,0}$ the growth period of disturbances to manifest convection becomes much shorter in comparison with that of the no-solute gradient.

ACKNOWLEDGEMENTS

The authors deeply appreciate the financial support from Yukong Limited, and also the Korea Science and Engineering Foundation. It is noted that most of this work was presented orally at the 22nd ICHMT International Symposium on Manufacturing and Materials Processing [Dubrovnik and Yugoslavia, 1990].

NOMENCLATURE

a : dimensionless horizontal wave number, $\sqrt{a_x^2+a_y^2}$
a* : modified wave number, $a\sqrt{\tau}$
C : concentration [(weight percent)]
D : differential operator with respect to η
g : gravitational acceleration [$m\ s^{-2}$]
L : depth of layer [m]
Le : Lewis number, α/α_s
Pr : Prandtl number, ν/α
Ra : thermal Rayleigh number, $g\beta\phi L^5/\alpha^2\nu$
Ra* : modified thermal Rayleigh number, $Ra\sqrt{\tau^5}$
Rs : solute Rayleigh number, $g\gamma L^3\Delta C/\alpha_s\nu$
Rs* : modified solute Rayleigh number, $Rs\tau^2$
T : temperature [K]
t : time [s]
w : z-component of dimensionless velocity
z : dimensionless vertical coordinate

Greek Letters

α : thermal diffusivity [m^2s^{-1}]
α_s : solute diffusivity [m^2s^{-1}]
β : volumetric thermal expansion coefficient [K^{-1}], $-(\partial\rho/\partial T)_C/\rho$
γ : volumetric solute expansion coefficient [(weight percent)$^{-1}$], $(\partial\rho/\partial C)_T/\rho$
δ : dimensionless thermal penetration depth, $\propto\sqrt{\tau}$
η : modified vertical distance, $z/\sqrt{\tau}$
θ : dimensionless temperature
ν : kinematic viscosity [m^2s^{-1}]
ρ : density [$Kg\ m^{-3}$]
τ : dimensionless time
ϕ : temporal rate of heating [Ks^{-1}]
Ψ : dimensionless concentration
∇^2 : dimensionless Laplacian
∇_1^2 : dimensionless horizontal Laplacian with respect to x and y

Subscripts

c : critical state
i : initial state
0 : basic state
1 : perturbed state

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